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Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let x, y be the sides and θ the included angle.

Then $\frac{1}{2}xy\ \sin\theta=$area, and $x+y=2a$.

$$\therefore \text{Average area}=\triangle=\frac{\frac{1}{2}\int_0^{2a}\int_0^{\pi}x(2a-x)\sin\theta\ dx\ d\theta}{\int_0^{2a}\int_0^{\pi}dx\ d\theta}.$$

$$\therefore \triangle=\frac{1}{4\pi a}\int_0^{2a}\int_0^{\pi}x(2a-x)\sin\theta\, dx\ d\theta=\frac{1}{2\pi a}\int_0^{2a}x(2a-x)dx=\frac{2a^2}{3\pi}.$$

169. Proposed by HENRY HEATON, Atlantic, Iowa.

What is the average length of all straight lines that can be drawn within a given square?

I. Solution by B. F. FINKEL, A. M., Fellow in The University of Pennsylvania.

Let $ABCD$ be the given square, the length of whose side is a; and let EF and $E'F'$ be two parallel lines: EF terminating in two adjacent sides of the square, and $E'F'$ in two opposite sides. Let AH, $=x$, be the perpendicular to EF from A, and let the angle $FAH=\theta$.

Then $EF=\dfrac{x}{\sin\theta\cos\theta}$, and $E'F'=\dfrac{a}{\cos\theta}$.

Since the law of distribution of the lines is not given, we will assume that those having a given direction are uniformly distributed, and those passing through a given point are distributed so that the number of lines lying in a given angle is directly proportional to the magnitude of the angle.

Now, the limits of x are 0 and $AH'=a\sin\theta$ for $EF=x/\sin\theta\cos\theta$, and $AH'=a\sin\theta$ and $AH''=a\cos\theta$, for $EF=E'F'=a/\cos\theta$; and the limits of θ are 0 and $\frac{1}{4}\pi$. Then the average length of lines drawn within the square is

$$l_{\triangle}=\frac{\int_0^{\frac{1}{4}\pi}d\theta\left[2\int_0^{a\sin\theta}\sec\theta\ \cos\theta\ xdx+\int_{a\sin\theta}^{a\cos\theta}a\sec\theta\ dx\right]}{\int_0^{\frac{1}{4}\pi}d\theta\int_0^{\sqrt{2}\,a\cos(\frac{1}{4}\pi-\theta)}dx}$$

$$=\frac{1}{a}\int_0^{\frac{1}{4}\pi}d\theta\left[2\int_0^{a\sin\theta}\sec\theta\csc\theta\ xdx+a\int_{a\sin\theta}^{a\cos\theta}\sec\theta\ dx\right]$$

$$=\frac{1}{a}\int_0^{\frac{1}{4}\pi}d\theta\left[\sec\theta\csc\theta\Big(x^2\Big)_0^{a\sin\theta}+a\sec\theta\Big(x\Big)_{a\sin\theta}^{a\cos\theta}\right]$$

$$=\frac{1}{a}\int_0^{\frac{1}{4}\pi}a^2\,d\theta=\tfrac{1}{4}\pi a.$$

II. Solution* by G. B. M. ZERR. A. M., Ph. D., Parsons, W. Va.

Let $ABCD$ be the given square. Let F be a point in AB, E a point in AD, and G, H, points in DC. Let PQ be a length, measured at random in FE and FG. Let $AF=x$, $AE=y$, $PQ=z$, $DH=u$, $DG=v$, $AB=a$, $\sqrt{(x^2+y^2)}=z'$, $\sqrt{[a^2+(u-v)^2]}=z''$. M=the average length.

$$\therefore M=\frac{\int_0^a\int_0^a\int_0^{z'} z\,dx\,dy\,dz+\int_0^a\int_0^u\int_0^{z''} z\,du\,dv\,dz}{\int_0^a\int_0^a\int_0^{z'} dx\,dy\,dz+\int_0^a\int_0^u\int_0^{z''} du\,dv\,dz}$$

$$=\frac{\int_0^a\int_0^a(x^2+y^2)dx\,dy+\int_0^a\int_0^u[a^2+(u-v)^2]du\,dv}{2\int_0^a\int_0^a\sqrt{(x^2+y^2)}dx\,dy+2\int_0^a\int_0^u\sqrt{[a^2+(u-v)^2}du\,dv}$$

$$=\frac{\int_0^a a(3x^2+a^2)dx+\int_0^a(3a^2u+u^3)du}{3\int_0^a\left[a\sqrt{(a^2+x^2)}+x^2\log\left(\frac{a+\sqrt{(a^2+x^2)}}{x}\right)\right]dx+3\int_0^a\left[u\sqrt{(a^2+u^2)}+a^2\log\left(\frac{u+\sqrt{(a^2+u^2)}}{a}\right)\right]du}$$

$$=\frac{11a}{4[2+\sqrt{2}+5\log(1+\sqrt{2})]}.$$

Also solved by J. E. Sanders.

171. Proposed by O. E. GLENN, A. M., Ph. D., Drury College.

There are n derelict steamers afloat in a circular sea of radius r. The water in the sea is moving northward in a current whose velocity varies inversely as the perpendicular distance from the north-south tangent to the sea on its west beach. Find the probability that a ship crossing the sea on a random diameter will encounter e derelicts during the voyage.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let O be the center of the sea, POQ the random diameter, G the position of a derelict when the start is made at P, AOB the diameter perpendicular to the north-south tangent AC on the west beach A. Let FE be the chord through G perpendicular to AB, cutting AB in L and PQ in K, H the foot of the perpendicular from Q on AB. Let $\angle QOB=\theta$, $AL=z$, $GK=y$, u=uniform velocity of ship on PQ, p=chance of meeting one derelict. Then $OK=OL\sec\theta=(z-r)\sec\theta$, velocity of derelict$=m/z$.

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*The two methods illustrated are typical of the different results to be obtained when different laws of distribution are assumed.